Formation Tracking for a Class of Uncertain Multi-Agent Systems: A Distributed Kalman Filtering Approach

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***Abstract*—Popular methods for formation tracking generally assume perfect local or relative state information. Where uncertainty is considered, local state estimates are obtained through a common communication network or via global observations from external sensors, leading to centralized, impractical, and unreliable control synthesis techniques. In light of these challenges, this letter presents a formation tracking scheme that relies on relative states estimated via distributed Kalman filtering (DKF). The DKF approach we employ utilizes the underlying shared network to reconstruct each agent's relative state from neighbor-wise relative state measurements. By adopting a leader-follower communication network topology and a role-based control scheme, we then apply the reconstructed states to synthesize distributed control inputs for the formation tracking task. We show formally that under standard observability and bounded uncertainty conditions, our approach contains the formation errors of the ensemble and ensures faithful trajectory tracking without inducing transient deformations, a common problem associated with conventional techniques. To further validate our method, we provide numerical results from formation tracking simulations of a single-integrator ensemble, which demonstrate that the DKF approach yields markedly smaller formation and tracking errors compared to standard Kalman filtering.**

***Index Terms*—Collective control, distributed Kalman filtering (DKF), formation tracking, leader-follower networks, multi-agent systems.**

I. Introduction

BROADLY speaking, formation tracking refers to the coordination of an ensemble of robots or autonomous vehicles to preserve a specific geometric configuration while tracking a reference path. Such autonomous coordination is critical to many real-world applications, such as aerial monitoring [1], wireless sensor networks [2], autonomous vehicle platooning, and near-Earth observation [3], among others. However, attaining accurate formation tracking is challenging since traditional formation tracking algorithms rely on reasonably accurate estimates of the agents' states. In practical applications, this challenge is amplified by measurement noise from both onboard and external sensors, significantly degrading the performance of the multi-agent system. To mitigate the effects of sensor noise, a popular approach is to employ Kalman filtering (KF), which entails reconstructing a dynamic state using local measurements from a *single* sensor. Conversely, in classical distributed Kalman filtering (DKF), the dynamic state is recovered using local measurements from a *group* of networked sensors. In other literature [4], DKF entails reconstructing the relative state of networked dynamic agents using relative sensor measurements. Although requiring several measurements, this distributed approach to KF-based estimation has been shown to be more robust to model uncertainty [5] than standard KF. It has also been applied successfully to solving a host of real-life estimation problems, ranging from cooperative localization to internal model average consensus estimation [6]. We refer the interested reader to [6] for an in-depth review on DKF.

Related Work: In many formation tracking articles, the subject of filtering is scarcely considered, as control signals are computed by assuming the agents' states are exact, or — in uncertain cases — that these states can be accessed from a shared communication network [7]. Such approaches to state estimation are, however, fickle and unreliable, since local measurements are rarely available to all agents and global sensing schemes are challenging to deploy in the wild. On the DKF front, fewer studies have developed formation tracking laws that require simultaneous state estimation (via DKF) and control. One such study is [4], which introduced a DKF-based formation control approach for a leader-follower (LF) network of single-integrator agents. This letter, however, focused on driving the agents to a specified formation without tracking a specific trajectory. The presentation in [8] considered both formation control and tracking (of an unknown leader trajectory). However, Kalman filtering was applied (without consideration

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of process noise) to reconstruct only the states of a real and a virtual (reference) leader in parallel.

Scientific Novelty: In this letter, we extend DKF-based results to the single leader formation tracking case, where the leader maintains its position while tracking a path setpoint while the followers focus solely on formation maintenance. Our work is distinct from [8], since we do not rely on a local Kalman estimator for reconstructing follower states with more than one neighbor, opting instead for the DKF technique described in Section II. We also uniquely account for both measurement noise and process disturbances in the formation tracking task, unlike [4], which considers only measurement noise. We also analyze the multi-agent network from two perspectives: a measurement graph dictates the sensor network, and a formation graph encodes the desired geometric pattern. We propose weighted consensus-like control laws for simultaneous formation maintenance and path following, yielding asymptotically-decaying errors. Lastly, we validate our approach using numerical simulations and compare our results with those obtained via standard Kalman filtering.

Notation: We denote vectors and matrices as x and A, with A^T denoting the transpose of A. We denote the Kronecker product as $\otimes$, the $d \times d$ identity matrix as I_d, and the vector $[\,\kappa,\ \kappa,\ \dots\ \kappa\,]^T$as $\kappa_d \in \mathbb{Z}^d$. The symbol $||x|| = [\,\sum_{i=1}^{d}([x]_i)^2]^{\frac{1}{2}}$ refers to the vector Euclidean norm, with $x \in \mathbb{R}^d$, and $\text{diag}([\,\star\,]_{i,i\in\mathcal{I}})$ is the $|\mathcal{I}|\times|\mathcal{I}|$ diagonal matrix, with $\mathcal{I}$ an index set, and where the i^{th} block, $[\,\star\,]_i$, can either be a scalar or a matrix. The set $\mathbb{R}_+$ denotes the set of positive real numbers, $\mathbb{Z}_+$ denotes the set of positive integers, and $\mathbb{E}$ is the expectation operator.

II. Preliminaries

To provide context and support for the theoretical proposition that follows, we present the following definitions and assumptions.

Definition 1 (DKF for Uncertain LTI Systems): Consider the following time-invariant stochastic difference equation

$$x_{t+1} = Ax_t + Bu_t + w_t \tag{1}$$

$$z_t = Cx_t + v_t,\ t \in \mathcal{T} := \{t \mid t = 0, 1, 2, \dots, T-1\}, \tag{2}$$

where $x \in \mathbb{R}^d$ is the system state, $u \in \mathbb{R}^m$ is the control input, $z \in \mathbb{R}^q$ is the measured output vector, and $T \in \mathbb{Z}_+$ is the problem time horizon. The matrices A, B, and C are the dynamics, input, and observation matrices of compatible dimensions, respectively. The vectors $w \sim \mathcal{N}(0, W)$ and $v \sim \mathcal{N}(0, V)$ denote independent and identically distributed (i.i.d.) Gaussian random variables representing the process noise and measurement noise, respectively, with corresponding covariance matrices, W and V, and zero cross-correlation for all values of t. Let Σ_t and Z_t respectively denote the error covariance at time t and the observation history up to time t. The estimated state corresponding to the Kalman observer with time-varying gain, L, is given by

$$\hat{x}_{t+1} = A\hat{x}_t + Bu_t + L_{t+1}\big(z_{t+1} - C(A\hat{x}_t + Bu_t)\big), \tag{3}$$

where $z_{t+1} - C(A\hat{x}_t + Bu_t)$ is the ***output prediction error***, which is Gaussian with zero mean, following standard Kalman filtering theory ([9], §8), and L is given by

$$L_{t+1} = \Sigma_{t+1}C^T\big(C\Sigma_{t+1}C^T + V\big)^{-1}. \tag{4}$$

In the DKF case, equations (1) and (2) are decoupled, so that (1), now indexed by i, represents the dynamic state (of agent i) to be estimated by n_i identical sensors, each maintaining a private observation of x_i governed by the measurement model

$$z_{j,t} = Cx_{i,t} + v_{j,t}, \quad j = 1, 2, \dots, n_i, \tag{5}$$

with distinct $v_j \sim \mathcal{N}(0, V_j)$. To adapt this definition to the present formation tracking setting, suppose the sensors are identical dynamic units (agents) instead, each fitted with the same sensor. Let each agent maintain a local measurement (evolving according to (5)) of its own state. Denote $z_{ij} = z_i - z_j$ as the relative output variable. We will assume that individual states, control inputs, and measurements are independent, so that, following [4] and (5), we can write the relative state measurement dynamics as

$$z_{ij,t} = Cx_{ij,t} + v_{ij,t}, \tag{6}$$

where $v_{ij,t} \sim \mathcal{N}(0, V_{ij})$ and $x_{ij} = x_i - x_j$ is the relative state. One can then design a Kalman filter to estimate x_{ij} from z_{ij} using (7) (the time argument has been suppressed for brevity)

$$\begin{aligned}\hat{x}_{ij,t+1} &= A\hat{x}_{ij,t} + B\big(u_{i,t} - u_{j,t}\big)\\ &\quad + L_{ij,t+1}\big(z_{ij,t+1} - C(A\hat{x}_{ij,t} + Bu_{i,t})\big).\end{aligned} \tag{7}$$

The Kalman gain thus becomes

$$L_{ij,t+1} = \Sigma_{ij,t+1}C^T\big(C\Sigma_{ij,t+1}C^T + V_{ij}\big)^{-1}, \tag{8}$$

with respective variables evolving according to

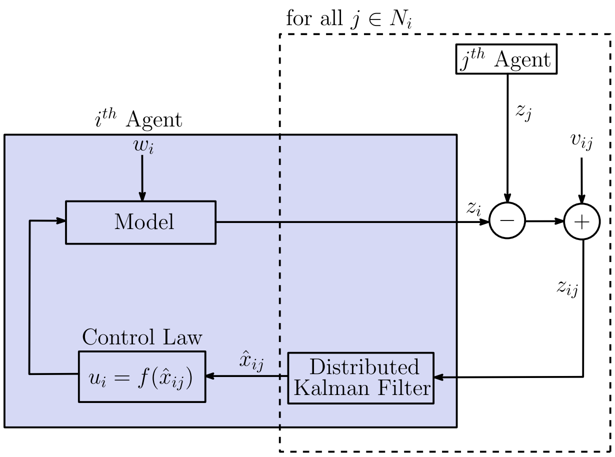
$$\left.\begin{aligned}\hat{x}_{ij,t} &= \mathbb{E}\big(x_{ij,t}|Z_{ij,t}\big)\\ \Sigma_{ij,t} &= \mathbb{E}\Big[\big(\hat{x}_{ij,t} - x_{ij,t}\big)\big(\hat{x}_{ij,t} - x_{ij,t}\big)^T\Big]\\ \Sigma_{ij,t+1|t} &= A\Sigma_{ij,t}A^T + W_{ij}\\ \hat{x}_{ij,t+1|t} &= \mathbb{E}\big(x_{ij,t+1}|Z_{ij,t}\big).\end{aligned}\right\} \tag{9}$$

One can thus write the relative state dynamics as follows

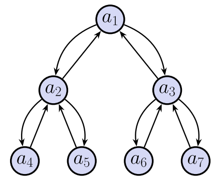
$$x_{ij,t+1} = Ax_{ij,t} + Bu_{ij,t} + w_{ij,t}, \tag{10}$$

where $w_{ij,t} \sim \mathcal{N}(0, W_{ij})$, and $u_{ij} = u_i - u_j \in \mathbb{R}^m$ is the forcing term for the relative state dynamics. A zero cross-correlation for V_{ij} and W_{ij} also applies in the relative case. Fig. 1a, partly adapted from [4], summarizes the DKF-based formation tracking concept considered in this letter.

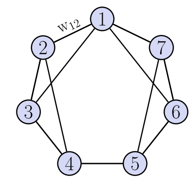
Definition 2 (Graph Theory): A ***directed*** and finite graph (hereafter, digraph) is the tuple, $\mathcal{D} = (\mathcal{V}, \mathcal{E})$, with $\mathcal{V}$ equal to the non-empty set $\{1, 2, \dots, n\}$ of n distinct elements, called ***nodes***, and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ equal to the ***edge*** set, $\{\{i, j\} \mid i, j \in \mathcal{V}\}$, where $\{i, j\} \in \mathcal{E} \implies \{j, i\} \notin \mathcal{E}$. The graph is ***weighted*** if there exists a map, $\mathrm{w} : \mathcal{E} \to \mathbb{R}$, that associates to each directed edge, a real number, denoted as w_{ij}. It is ***unweighted*** otherwise. The digraph is ***strongly connected*** if there exists a ***directed path*** between any two nodes in $\mathcal{V}$. We will use the term ***arborescent*** to describe a digraph containing a **spanning rooted out-branching tree** (ROT). An ROT is a directed tree that spans $\mathcal{D}$ and includes directed paths from parent to child nodes in the tree. Finally, we denote the ***in-neighborhood***



(a) DKF-based formation tracking control block diagram: z_{ij} and $\hat{x}_{ij}$ are the relative output measurement and the estimated relative state between agents i and j. The i^{th} control law, u_i, is a function of the estimated relative state.



(b) An unweighted and strongly connected digraph on seven vertices illustrating the leader-follower measurement network structure under consideration. The symbol a_i denotes the i^{th} agent, with a_1 indicating the leader.



(c) Weighted formation graph on seven vertices. To preserve the legibility of the figure, only one edge has been labeled for reference.

Fig. 1. (a) DKF-based formation tracking control block diagram: z_{ij} and $\hat{x}_{ij}$ are the relative output measurement and the estimated relative state between agents i and j. The $\imath^{th}$ control law, u_i, is a function of the estimated relative state. (b) An unweighted and strongly connected digraph on seven vertices illustrating the leader-follower measurement network structure under consideration. The symbol a_i denotes the $\imath^{th}$ agent, with a_1 indicating the leader. (c) Weighted formation graph on seven vertices. To preserve the legibility of the figure, only one edge has been labeled for reference.

and ***out-neighborhood*** sets of the i^{th} node as N_i^{in} and N_i^{out}, respectively.

Definition 3 (Formation Control): We define the **measurement graph** (Fig. 1b) as the unweighted digraph, $\mathcal{D}_m = (\mathcal{V}, \mathcal{E}_m)$, with edge set, $\mathcal{E}_m$, where $\{j, i\} \in \mathcal{E}_m$ implies that z_{ij} is available to agent i. The **formation graph** (Fig. 1c), $\mathcal{G}_f = (\mathcal{V}, \mathcal{E}_f, \mathrm{W})$, is a weighted and spanning subgraph of the disoriented digraph ($\tilde{\mathcal{G}} = (\mathcal{V}, \mathcal{E})$) that governs the agents' interaction, with edge set, $\mathcal{E}_f \subseteq \mathcal{E}(\tilde{\mathcal{G}})$, and where W is a weight matrix encoding the desired pairwise distances between agents i and j, given by the specification, $D = \{d_{ij} \in \mathbb{R}_+ | \; d_{ij} = d_{ji} > 0\}$. Given that $\mathcal{G}_f$ is undirected, a set, N_i, denoting the i^{th} node's neighborhood (without considering edge direction) applies. The formation is ***feasible*** if every $d_{ij} \in D$ is chosen so that the formation makes sense geometrically. That is, there exists ξ_i and ξ_j in $\mathbb{R}^d \ni ||\xi_i - \xi_j|| = d_{ij} \; \forall \; i, j \in \mathcal{V}$. A formation is ***rigid*** if all pairwise distances remain constant for all time ($||\xi_{i,t} - \xi_{j,t}|| = d_{ij} \; \forall \; t, \; \{i, j\} \in \mathcal{V} \times \mathcal{V}$); it is ***minimally rigid*** if the removal of a single edge renders the formation ***non-rigid*** ([10], §6). For $d = 2$, minimal rigidity reduces to the condition: $|\mathcal{E}_f| = 2|\mathcal{V}| - 3$.

III. Problem Description

We consider the formation tracking problem — comprising formation maintenance ($\mathcal{P}_d$ (a)) and leader trajectory tracking ($\mathcal{P}_d$ (b)) — and defined formally as follows:

($\mathcal{P}_d$: (a) Given the leader agent's reference trajectory, $x^r_{1,t}$, and taking d_{ij} to be the desired pairwise separation defined by $\mathcal{G}_f$, find a control, u_i, for the i^{th} agent in the ensemble that ensures that the estimated relative state, $\hat{x}_k$, corresponding to the k^{th} edge ($k = 1, 2, \ldots, |\mathcal{E}_f|$) between nodes i and j (with $i, j \in \mathcal{V}$) satisfies

$$\|\hat{x}_k\| = \|\hat{x}_{i,t} - \hat{x}_{j,t}\| = d_{ij}, \forall \; j \in N_i, \forall \; t \in \mathcal{T};$$

(b) Find a control, $u_{1,t}$, that (simultaneously) drives the leader so that $\lim_{t\to\infty} \|\hat{x}_{1,t} - x^r_{1,t}\| = 0$.

We study $\mathcal{P}_d$ under the following assumptions:

($\mathcal{A}_1$) The formation specified by the formation graph is feasible, invariant to homogeneous transformations in $SE(2)$, and minimally rigid (see Definition 3).

($\mathcal{A}_2$) The measurement graph is uniformly arborescent (see Definition 2), i.e., $\exists$ a $\Delta T < \infty$ such that, from t to $t + \Delta T$, $\mathcal{D}_m$ contains an ROT $\forall \; t$.

($\mathcal{A}_3$) There exists a stabilizing feedback law, $\phi : \mathbb{R}^{m\times d} \times \mathbb{R}^d \to \mathbb{R}^m$, belonging to the set of piecewise continuous and absolutely-integrable functions in $\mathbb{R}^m$ that are stabilizing. That is, $\rho(A - BK_\phi) < 1$, where K_ϕ and ρ are the control gain matrix and the spectral radius of the closed-loop system associated with ϕ.

($\mathcal{A}_4$) The measurement model (2) is stochastically-observable, in the sense of [11], so that the error covariance is bounded.

($\mathcal{A}_5$) There exists a positive definite matrix $P \in \mathbb{R}^{d\times d}$ satisfying

$$(A - I_d)^T P (A - I_d) + K_{u_k}^T B^T P B K_{u_k} \preceq 0, \quad (11)$$

where K_{u_k} is a controller gain matrix of appropriate dimension corresponding to u_k, i.e., $u_k = -K_{u_k}\hat{x}_k$.

Remark 1 (Justification for Assumptions): In the aforementioned assumptions, Assumption $\mathcal{A}_1$ is a sufficient condition that guarantees the convergence of a single-integrator ensemble to any desired formation under forcing terms specified by the agreement protocol [12], while Assumption $\mathcal{A}_2$ is needed to guarantee uniform connectivity, a condition necessary for directed switched consensus ([10], §4). Assumption $\mathcal{A}_3$ is required to argue for the asymptotic stability of the formation tracking law. Assumption $\mathcal{A}_4$ is a common requirement for making precise claims regarding the decay of the output prediction error and corresponds to the requirement that the measurements are sufficiently informative to accurately predict the actual system state. Finally, Assumption $\mathcal{A}_5$ is needed to prove — via Lyapunov arguments — that the formation tracking law (13) is asymptotically stabilizing. Admittedly, without persistent network connectivity, such stabilization is not possible. In the time-varying directed network case, this means that the measurement graph must contain an ROT sufficiently often (Assumption $\mathcal{A}_2$).

IV. Main Result: DKF-Based Control Synthesis for Formation Tracking

Suppose we deploy a team of agents for the formation tracking task, where inter-agent interactions are dictated by a leader-follower network (encoded by a digraph), with the state of the i^{th} agent evolving according to the dynamics

$$x_{i,t+1} = Ax_{i,t} + Bu_{i,t} + w_{i,t}, \tag{12}$$

and with sensor models following (5). A natural question one might ask is the following: what conditions are required to guarantee acceptable execution of the formation tracking task for the described LF network? The following proposition provides some insight.

Proposition 1: Under Assumptions $\mathcal{A}_1$ to $\mathcal{A}_5$, with each $\hat{x}_{ij}$ evolving according to (7), the control law

$$u_{i,t} = \begin{cases} -\sum_{j\in N_i} w^E_{ij}\hat{x}_{ij,t}, & i \neq 1 \\ \left[-\sum_{j\in N_i} w^E_{ij}\hat{x}_{ij,t}\right] + \phi\left(\hat{x}_{i,t} - x^r_{i,t}\right), & i = 1, \end{cases} \tag{13}$$

will result in convergence to the desired formation with simultaneous asymptotic tracking of the reference trajectory ($x^r_{i,t}$, $i = 1$), even with possibly noisy sensor measurements. w^E_{ij} and $\phi(\cdot)$ are a time-varying weight and a tracking law for the leader, given in Sections IV-A and IV-B, respectively.

Proof: See Appendix A. ■

Remark 2: In addition to a path-following component, the leader's control law in (13) comprises a formation-keeping component, which ensures that it is slowed down appropriately while it tracks its reference trajectory, so that its velocity matches that of the followers to prevent transient deformations. Also note that, while the proof in Appendix A considers a single-integrator ensemble, the arguments there can be extended to the general linear time-invariant case under the aforementioned assumptions and provided that the noise statistics satisfy the conditions given in Definition 1.

A. Formation Maintenance via Edge Tension Energy Minimization

To determine the weights, consider a potential-like function of the relative state vector norm, $E_{ij}(||x_i - x_j||)$, which characterizes the virtual (edge tension) energy required to keep agents i and j connected within the communication network. Agent-level controller input signals can then be synthesized from this energy function by solving for controls that minimize the total edge tension energy, $E = \frac{1}{2}\sum_{i=1}^n \sum_{j=1}^n E_{ij}$. An obvious choice is to command inputs equal to $-\nabla^T_{x_i} E$, the negative gradient of E with respect to x_i. As already established in several studies (see [10, Sec. 8], for one instance), such a control strategy leads directly to the following weighted distributed control law

$$\dot{x}_i = \sum_{j\in N_i} w^E_{ij}(x_j - x_i), \tag{14}$$

where the weights,[1] w^E_{ij}, are obtained from E_{ij} using $(1/s)(\partial E_{ij}/\partial s)|_{s=||x_i - x_j||}$. Consider now the following candidate edge tension function, which attains a global minimum at $\hat{x}_{ij} = d_{ij}$:

$$E_{ij}\left(||\hat{x}_{ij}||\right) = \begin{cases} \left(||\hat{x}_{ij}|| - d_{ij}\right)^2, & \{i,j\} \in E_f \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

We can write the corresponding weights from (15) as

$$w^E_{ij} = \begin{cases} \frac{k(||\hat{x}_{ij}|| - d_{ij})}{||\hat{x}_{ij}||}, & \{i,j\} \in E_f \\ 0, & \text{otherwise,} \end{cases} \tag{16}$$

where the time argument has been suppressed for brevity. k is a positive scaling factor. Note that, as is characteristic of potential-like functions and from (15) and E's definition, it follows that: E is continuously-differentiable, with $E(d_{ij}) = 0$, and $E(||\hat{x}_{ij}||)$ is positive for $||\hat{x}_{ij}|| \neq d_{ij}$.

B. Leader Trajectory Tracking Law

To determine $\phi(\cdot)$, we solve for the control sequence, $\{u_{1,t}\}_{t=0}^{T-1}$, that satisfies the following optimization problem

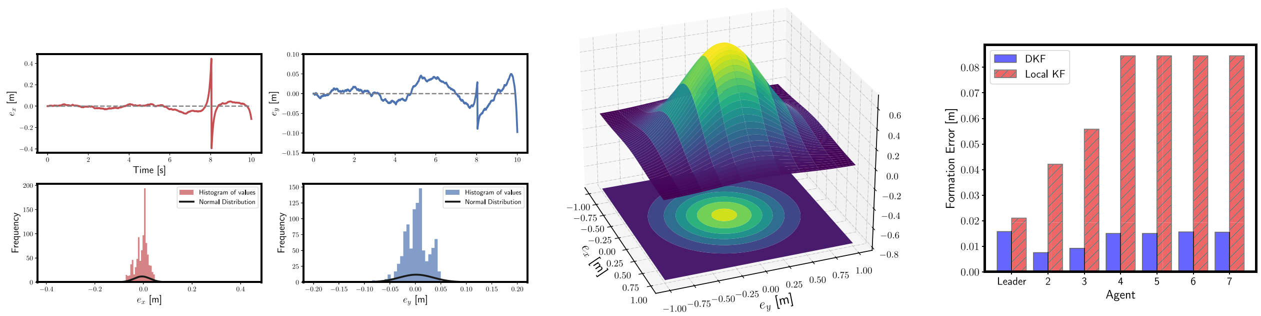
$$\min_{u_1} \sum_{t=0}^{T-1} u^T_{1,t} R u_{1,t} + \left(x_{1,t} - x^r_{1,t}\right)^T Q \left(x_{1,t} - x^r_{1,t}\right)$$

$$\text{s.t.:} \quad (3)\Big|_{x_{1,t}=\hat{x}_{1,t}, u_t = u_{1,t}} \tag{17a}$$

$$x_{1,0} = \hat{x}^0_1, -u_{1\max} \leq u_{1,t} \leq u_{1\max} \ \forall\ t \in \mathcal{T}, \tag{17b}$$

where $\hat{x}^0_1$ is the leader's initial estimated state, $u_{1\max} \in \mathbb{R}^m$ is a vector of control bounds, $R \succ 0$, and $Q \succeq 0$. $\hat{x}_1$ is obtained via the leader's Kalman estimator, following Eqs. (3) and (4), and the optimal control law is then subsequently computed from the estimated state by solving (17). Note that the quadratic criterion as well as the equality (17a) and linear (each component of (17b)) constraints guarantee the existence of a solution (ϕ) satisfying Assumption $\mathcal{A}_3$. Similar in spirit to the separation principle in systems and control theory, the DKF formation tracking scheme works by performing estimation using unforced dynamics and then synthesizing control inputs via the reconstructed states. We provide a simple algorithm (Algorithm 1) summarizing the foregoing ideas.

[1]These edge tension weights are distinct (and have been typeset with an E superscript to distinguish them) from the weights of the formation graph that encode the desired pairwise distances, w_{ij}, and the relative process noise, w_{ij}.



(a) (b) (c)

Fig. 2. (a) Top row: Plots of the (component-wise) time evolution of the trajectory error ($e = [e_x, e_y]^T$) for the leader agent, with horizontal dashed lines at $e_x = 0$ and $e_y = 0$ for reference. Bottom row: A corresponding histogram compared against a bivariate Gaussian (in black) generated from the mean and covariance of 1000 leader trajectory error samples. (b) Bivariate Gaussian distribution over 1000 leader trajectory error samples. (c) Formation error for all agents compared according to state estimation technique. The DKF technique clearly outperforms standard Kalman filtering.

Algorithm 1 DKF-Based Controller Synthesis for Formation Tracking

Require: T, N, Relative state dynamics: A, B, C.
Relative noise covariance matrices: W_{ij}, V_{ij}.
$\mathcal{D}_m$, $\mathcal{G}_f$, and associated matrix objects.
R, Q, $u_{1\max}$, d_{ij}.
Compute $\hat{x}_1$ and solve (17) for $\phi(\cdot)$.
for $\tau \leftarrow 0$ to $T-1$ **do**
  for $i \leftarrow 1$ to N, $j \in N_i^{in}$ **do**
    Compute $L_{ij,\tau+1}$ (8), $\hat{x}_{ij,\tau}$ (9), $w_{ij,\tau}^E$ (16), $u_{i,\tau}$ (13).
  end for
  $\hat{x}_{ij,\tau+1} \leftarrow$ rhs (7)[3].
end for

V. Simulation Example

Consider a group of n agents in the plane, each represented by single-integrator dynamics subject to noisy inputs and with noise-corrupted measurements, as in

$$x_{i,t+1} = x_{i,t} + \delta_t u_{i,t} + w_{i,t} \tag{18}$$

$$z_{i,t} = x_{i,t} + v_{i,t}, \quad i = 1, 2, \ldots, n, \tag{19}$$

where each $x_i \in \mathbb{R}^2$, $u_i \in \mathbb{R}^2$, $w_i \sim \mathcal{N}(0, \sigma_i^2 I_2)$, $v_i \sim \mathcal{N}(0, \rho_i^2 I_2)$, $\rho_i > \sigma_i > 0\ \forall\ i = 1, 2, \ldots, n$, and $\delta_t > 0$ is a sampling interval. The leader is required to track a feasible trajectory, and all agents share a common sense of orientation. We set each σ_i and ρ_i to the values given in the last two columns of Table I[2] and construct $\mathcal{G}_f$ using the set of points, p_i, in the plane, given by $[\cos(\pi - 2\pi\frac{(i-1)}{n}), \sin(\pi - 2\pi\frac{(i-1)}{n})]^T$ and depicted on Figure 1(c). Clearly, G_f is minimally rigid. It is also easy to verify that $\Delta^{in}(\mathcal{D}_m)$ is equal to $\mathrm{diag}([\,2,\ 2,\ 2,\ 1,\ 1,\ 1,\ 1\,])$. Other matrix objects have been omitted for brevity.

VI. Results & Discussion

Table I summarizes the numerical results from formation tracking simulations with the seven-agent collective. Here, we compare root-mean-squared-error (RMSE) values of the tracking (leader) and formation (leader and follower) errors for all agents, under the DKF and standard KF techniques. While the tracking RMSE measures the deviation of the leader's trajectory from the given reference, the formation RMSE assesses the difference between the desired inter-agent distances and the actual distances observed between the agents. On the whole, these errors are relatively small compared to the regular KF estimation results (Fig. 2c), indicating good formation tracking performance under the DKF-based scheme. However, we also observe that although the leader tracks its reference satisfactorily, its formation maintenance (in the DKF case) is not on par with that of the followers, evident from its perceptibly higher formation RMSE. In the literature, a converse trend has been reported when the leader is instead driven by a tracking-only control law, resulting in almost-precise trajectory tracking with intermediate deformations [13]. Such inter-task trade-offs are to be expected in scenarios involving multiple agents with possibly competing objectives.

TABLE I
Tracking and Formation Errors (RMSE)

Agent (i)	RMSE Tracking	RMSE Formation		σ_i	ρ_i
1	0.018	0.01574	0.021	0.01	0.05
2	–	0.007	0.0421	0.05	0.1
3	–	0.009	0.0558	0.05	0.08
4	–	0.015	0.0845	0.03	0.15
5	–	0.015	0.0845	0.06	0.2
6	–	0.0156	0.0845	0.06	0.9
7	–	0.0155	0.0845	0.02	0.12
Estimation Technique		DKF	KF		

VII. Conclusion

This letter proposed a control technique for formation tracking under process and measurement uncertainty by coalescing ideas in distributed Kalman filtering and leader-follower consensus. We showed that, under standard system observability, bounded uncertainty, and network connectivity conditions, it is possible to obtain satisfactory geometry preservation with multi-agent path following by commanding agent-level weighted control laws synthesized from relative state estimates. Synthesizing distributed control inputs using

[2]Our choice to have higher measurement noise than process noise is completely arbitrary.

[3]rhs($\star$) is a placeholder function that evaluates and returns the expression on the right-hand side of $\star$.

similar ideas, but under other practical constraints, such as an obstacle-rich mission space or a non-uniformly connected communication network topology, remain open and are good candidates for future extensions of the foregoing work.

Appendix
Proof of Proposition 1

Proof: For simplicity and as a motivating instance, let the agents evolve according to the discrete-time single-integrator model in (18). Dropping the time argument for conciseness, define the formation error state of the system as $e_k = \hat{x}_k - p_k$, $k = 1, 2, \ldots, \ell$, where $\ell = |\mathcal{E}_f|$, the number of edges in the formation graph, and $p_k = p_i - p_j, i \neq j$, is a vector capturing the given formation specification, that is, $||p_i - p_j|| = d_{ij}$. Let the vectors corresponding to the formation error state, estimated relative state, and formation specification state for the ensemble system be respectively denoted by $e = [e_1^T, e_2^T, \ldots, e_\ell^T]^T \in \mathbb{R}^{\ell d}$, $\hat{x} = [\hat{x}_1^T, \hat{x}_2^T, \ldots, \hat{x}_\ell^T]^T \in \mathbb{R}^{\ell d}$, and $p = [p_1^T, p_2^T, \ldots, p_\ell^T]^T \in \mathbb{R}^{\ell d}$. With this notation and from (13), we can write the ensemble-level control law for the relative state dynamics in the compact form,

$$u = -\tilde{W}^E \hat{x}, \tag{20}$$

where u is the ensemble-level relative-state control vector ($u_{ij} = u_k$) given as $u = [u_1^T, u_2^T, \ldots, u_\ell^T]^T \in \mathbb{R}^{\ell d}$. $\tilde{W}^E$ is the diagonal weight matrix $\mathrm{diag}([(\tilde{W}_k^E \otimes I_d)|\ k = 1, 2, \ldots, \ell]) \in \mathbb{R}^{\ell d \times \ell d}$, with $\tilde{W}_k^E$ denoting the edge tension weight matrix corresponding to the k^{th} component of u and given by

$$\tilde{W}_k^E \in \mathbb{R}^{\ell \times \ell} = \begin{cases} w_{ih}^E, & \text{if } h \in N_i \\ -w_{jf}^E, & \text{if } f \in N_j \\ 0, & \text{otherwise.} \end{cases} \tag{21}$$

Writing the ensemble-level formation error dynamics as $e^+ = e + \hat{x}^+ - \hat{x}$, where, for convenience, we have denoted the ensemble state one time-step into the future with a "+" superscript, we select the following Lyapunov function candidate

$$V(e) = e^T (I_\ell \otimes P) e, \tag{22}$$

where $P \in \mathbb{R}^{d \times d}$ is a positive definite matrix. Note that $V = 0 \iff e = 0_{\ell d}$, or, equivalently, $\iff \hat{x}_k = p_k\ \forall\ k$. However, since V is stochastic owing to its dependence on $\hat{x}$, to determine if V decreases along trajectories of the closed-loop system, we have to check that the *expected* difference

$$\begin{aligned} \mathbb{E}\big(V\big(e^+\big) - V(e)\big) &= \big(e^+ - e\big)^T (I_\ell \otimes P)\big(e^+ - e\big) \quad &(23)\\ &= \big(\hat{x}^+ - \hat{x}\big)^T (I_\ell \otimes P)\big(\hat{x}^+ - \hat{x}\big), \quad &(24) \end{aligned}$$

is non-positive. Consider the right-hand side of (24). The expression corresponding to the k^{th} edge is

$$\begin{aligned} \big(\hat{x}_k^+ - \hat{x}_k\big)^T P\big(\hat{x}_k^+ - \hat{x}_k\big) &= ((A - I_d)\hat{x}_k \\ &\quad + Bu_k + L_k(z_k - C\hat{x}_k))^T P((A - I_d)\hat{x}_k \\ &\quad + Bu_k + L_k(z_k - C\hat{x}_k)) \\ &= \hat{x}_k^T (A - I_d)^T P(A - I_d)\hat{x}_k + u_k^T B^T P B u_k \\ &\quad + \big(z_k - C\hat{x}_k\big)^T L_k^T P L_k\big(z_k - C\hat{x}_k\big) \\ &= \hat{x}_k^T (A - I_d)^T P(A - I_d)\hat{x}_k \\ &\quad + \hat{x}_k^T \big(\tilde{W}_k^E \otimes I_d\big)^T B^T P B\big(\tilde{W}_k^E \otimes I_d\big)\hat{x}_k \\ &\quad + \big(z_k - C\hat{x}_k\big)^T L_k^T P L_k\big(z_k - C\hat{x}_k\big) \\ &= \hat{x}_k^T \big[(A - I_d)^T P(A - I_d) \\ &\quad + \big(\tilde{W}_k^E \otimes I_d\big)^T B^T P B\big(\tilde{W}_k^E \otimes I_d\big)\big]\hat{x}_k \\ &\quad + \big(z_k - C\hat{x}_k\big)^T L_k^T P L_k\big(z_k - C\hat{x}_k\big). \end{aligned} \tag{25}$$

Since $z_k - C\hat{x}_k$ is Gaussian with zero mean, and by the additive property of the expected value, it follows that

$$\begin{aligned} \mathbb{E}(V(e^+) - V(e)) = \hat{x}_k^T \big[&(A - I_d)^T P(A - I_d) \\ &+ (\tilde{W}_k^E \otimes I_d)^T B^T P B(\tilde{W}_k^E \otimes I_d)\big]\hat{x}_k \leq 0, \end{aligned} \tag{26}$$

under Assumption $\mathcal{A}_5$, with $K_{u_k} = \tilde{W}_k^E \otimes I_d$. Hence, $V(e)$ is a common weak Lyapunov function for the system. By LaSalle's invariance principle, $\hat{x}$ will converge to the largest invariant set within the set, $V_0 = \{e \mid \mathbb{E}(V(e^+) - V(e)) = 0\}$, i.e., the set $\mathcal{M} \subseteq V_0$ for which $e = \vartheta \implies \vartheta + \hat{x}^+ - \hat{x} = \vartheta$. It can be shown that $\mathcal{M} = \{e | \mathbb{E}(e_k) = 0\ \forall\ k\}$. As such, $\hat{x}_k$ will converge to $p_k\ \forall\ k$, thus maintaining the formation. Moreover, under Assumption $\mathcal{A}_3$, the leader will be driven so that its state asymptotically converges to the target state. As such, the collective will satisfy the formation tracking requirement, which completes the proof. ■

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