



Research article

Robust control of a DC motor

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ABSTRACT

This paper presents an observer-based active disturbance rejection control (ADRC) structure which achieves robust tracking performance, effective regulatory control, and negligible internal uncertainty for a separately-excited DC motor. The approach is primarily founded on the opportunity of online estimation of all the disturbance inputs – known and unknown – which affects a DC motor utilizing suitable observers, and proceeding to reject them through an appropriate feedback control law. The critical tuning parameters for this controller are the observer and controller bandwidths; the higher values of which will result in enhanced disturbance-rejection performance. A frequency response analysis demonstrates that the observer-based controller significantly improves the robustness of the DC motor to external disturbances, and remarkably maintains its bandwidth and stability margin regardless of the unknown dynamics and parametric variations existing in the motor. A thorough comparison is made between the proposed control strategy and previous model-based strategies used via control simulations to validate its effectiveness.

1. Introduction

DC motors are a common choice for various industrial applications where servo-control is necessary [1, 2, 3, 4]. A recurring control objective for the machine is to accurately regulate its speed in the incidence of parametric changes (modelling uncertainty), unknown dynamics, and external disturbances. Separately-excited direct-current motors (SEDCM) refer to a class of direct-current (DC) machines whose armature and field circuits are supplied from independent voltage sources. Armature-based speed control of DC motors is well documented in the literature [1, 2, 3, 4]. Modelling uncertainty is significant in this machine because of the difficulty in accurately describing a complex nonlinear system.

1.1. Contribution to knowledge

Theoretical and practical control agrees that the proportional-integral-derivative (PID) controller is the most widespread and ubiquitous in industry; nevertheless, it has significant limitations [5, 6]. This article, however, utilizes an active disturbance rejection control (ADRC) scheme – whose strength lies in the design of a well-behaved observer – to regulate the speed, improve the robustness and stability margin of the DC motor [6, 7, 8].

Similarly, an exhaustive frequency response analysis of the motor using Bode plots is presented in this work, to demonstrate how the

observer-based controller dramatically improves the robustness of the DC motor to disturbances and unknown dynamics, and remarkably maintains its bandwidth and stability margin regardless of parametric variations.

A comparative study will be made between the proposed observer-based controller, the feedback-feedforward compensator proposed in [1], and the classical PID control scheme. This study will help to portray further the radical departure from the traditional model-based feedback control paradigm, presented by the observer-based controller.

1.2. Background

This observer-based control scheme does not need a precise model of the motor for effectiveness; it instead uses intelligent heuristic techniques and basic knowledge of the workings of a DC motor to actively estimate and compensate the disturbances affecting the machine.

The controller not only produces desired performance characteristics but also cancels disturbances and model uncertainties in real-time, all while retaining the tuning simplicity and error-driven framework of PID control [6, 8]. The Extended State Observer (ESO) reduces the time-varying DC motor to a much simpler form – a well-behaved plant – by actively estimating and rejecting the generalized disturbance in the inner loop so that the state feedback controller can carry out nominal control in the outer loop.

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As outlined in [6, 9], the ADRC paradigm fundamentally involves utilizing an augmented state-space model of the plant, which contains the generalized disturbance as an additional state. This other state approximates the disturbance. Typically, it is expected that the ESO will use nonlinear observer gains, however, for simplicity of tuning and controller deployment, linear gains are used instead and parameterized as in [8], making the observer bandwidth the only tuning parameter. With a well-behaved observer, added simplification of the controller tuning is achieved by making its parameters, functions of the closed-loop bandwidth [9]. This observer-based ARDC approach heralds a crucial transition to modern control and is gradually replacing the widespread classical PID control and the more recent model-based solutions among practitioners [10].

Considerable effort has been devoted to the motion control of the separately-excited DC motor using model-based control techniques. Still, these efforts have not been efficient in canceling out external disturbances and improving its robustness to parametric uncertainties [1, 2, 3]. Reference [11] utilized linear model-predictive control (MPC) to regulate the speed of a DC motor. However, the machine's stability and response to parametric changes are not portrayed. The speed of the DC motor is adjusted via an IMC-based PI-like controller in [12]. This approach reduces the integral absolute error (IAE), but renders the system prone to oscillations, and hence, instability. ADRC technique is utilized in [13] to control the position of a DC motor; however, frequency analysis is not used to provide insight to its ability to withstand parametric uncertainties.

The rest of this paper is organized as follows: modelling and observer-based control of the machine is presented in Section II. Section III features the simulations and results, as well as the frequency analysis and deductions. Finally, concluding notes and recommendations for further research are given in Section IV.

2. Modeling and control

The dynamic model of the SEDCM is well detailed in literature [1, 2, 3, 14, 15, 16, 17, 18, 19]. The transfer function representing the dynamic relationship between the motor speed ω (the output y), armature voltage V_a and load torque T_d is given in (1):

$$\omega(s) = G_m V_a + G_d T_d \quad (1)$$

where G_m shows the dynamic relationship between the motor speed ω and the armature voltage V_a , while G_d depicts the dynamic relationship between the motor speed ω and load torque – the disturbance. Thus, (1) is rewritten to show a second-order dynamic relationship in (2) – assuming that the load torque applied to the DC motor takes a scalar nominal value:

$$\omega(s) = \frac{b}{s^2 + a_1 s + a_0} V_a - T_d \quad (2)$$

The DC motor is a linear time-invariant second-order plant:

$$\ddot{y} = -a_1 \dot{y} - a_0 y + b u \quad (3)$$

with a_0 and a_1 unknown, $h = -a_1 \dot{y} - a_0 y$ in this case. The DC motor, which in this case, is a second-order system is depicted thus

$$\ddot{y} = h(y, u, d) + b u \quad (4)$$

where b is the critical gain of the plant which represents the power with which the control output u can influence the controlled variable y , y is the output, u is the input, and h is the generalized disturbance consisting of unknown internal dynamics and external disturbances. Constructing an augmented state variable, where $\dot{h}$ is the augmented/extended state, in (5) such that the generalized disturbance for the DC motor is captured; and letting $z_1 = y$, $z_2 = \dot{y}$, $z_3 = h$ and assuming h is differentiable, (5) takes the form:

$$\dot{z} = A z + B u + E \dot{h} \quad (5)$$

$$y = C z \quad (6)$$

Where:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}, E = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$C = [1 \ 0 \ 0]$ and $z = [z_1 \ z_2 \ z_3]^T$. With the proposed approach, a well behaved Linear Extended State Observer (LESO) for the augmented model is designed, given by

$$\dot{\hat{z}} = A z + B u + L(y - \hat{y}) \quad (7)$$

$$\hat{y} = C \hat{z} \quad (8)$$

The observer gain vector L is chosen such that all the poles of the ESO are located at $-\omega_o$, that is, the idea is to place all the poles of the ESO at the same location. Which is derived with the transfer function from the output y to the control signal u as thus:

$$\lambda = (s + \omega_o)^3 = s^3 + \beta_1 s^2 + \beta_2 s + \beta_3$$

$$\Rightarrow \beta_1 = 3\omega_o, \beta_2 = 3\omega_o^2, \beta_3 = \omega_o^3$$

The gain vector of the state observer is given thus:

$$L = [3\omega_o \ 3\omega_o^2 \ \omega_o^3]^T \quad (9)$$

With a well-behaved observer, z_1 , z_2 and z_3 closely track y , $\dot{y}$, and h respectively. The observer is designed such that the Observer Tracking Error $e = (A - LC)e$ is asymptotically stable as time goes to infinity and its derivative, $\dot{e} = (A - LC)e$ is the Hurwitz. The ADRC controller then utilizes the estimated general disturbance z_3 in control to reject the

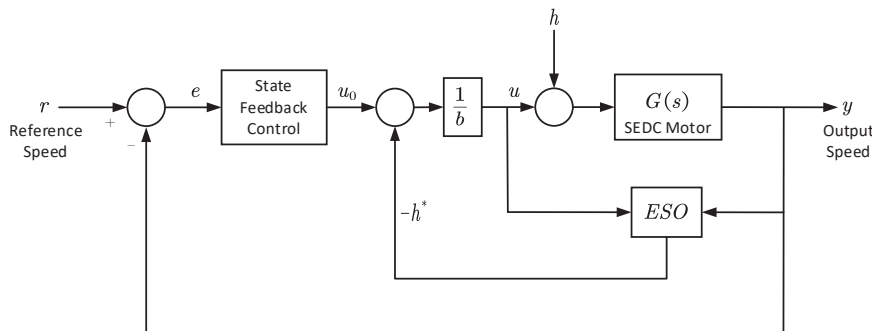


Figure 1. Observer-based ADRC control structure.

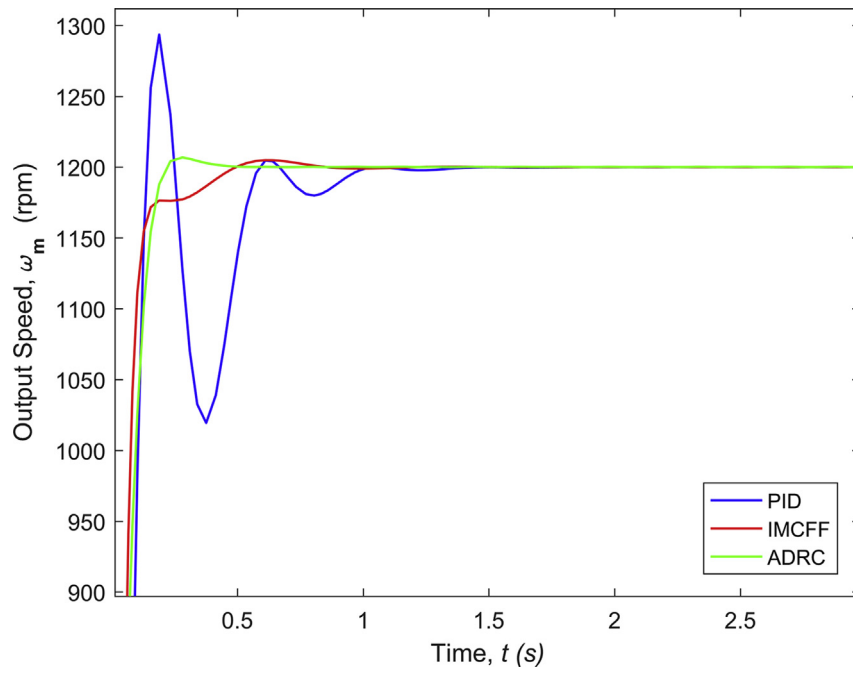


Figure 2. Robust Tracking Performance of the ADRC controller.

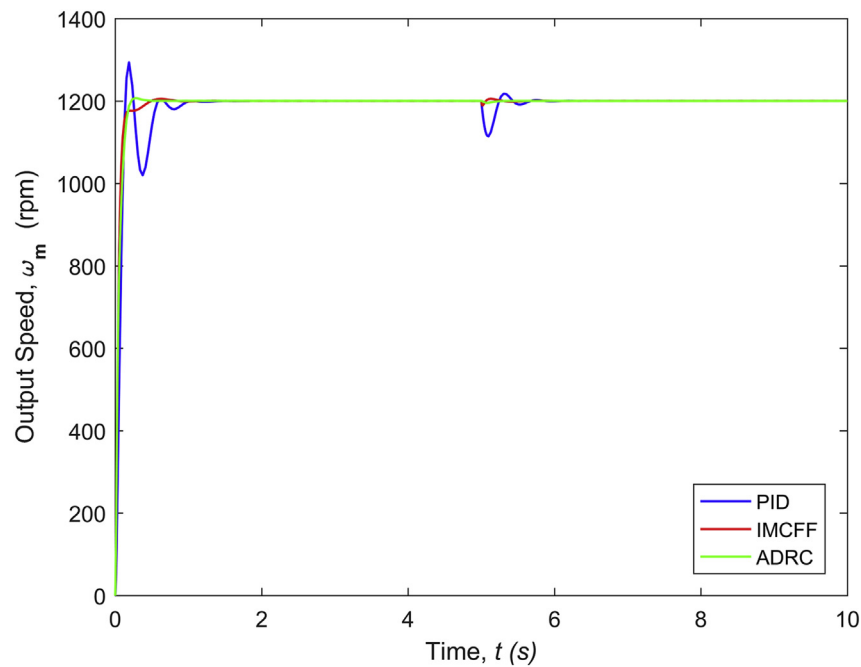


Figure 3. Regulatory Control Performance of the ADRC controller.

disturbance as in the resulting state-feedback law. This relationship is given by a Proportional-Derivative (PD) controller derived via the state-feedback rule [9, 20].

$$u = K \cdot [(r - z_1), (\dot{r} - z_2), -z_3] = \frac{k_p(r - z_1) + k_d(\dot{r} - z_2) - z_3}{b} \quad (10)$$

$$u_0 = k_p(r - z_1) - k_d z_2 \quad (11)$$

where r is the reference signal, $\dot{r}$ is its derivative, and K is the gain vector of the controller given by:

$$K = \frac{[k_p, k_d, 1]}{b} \quad (12)$$

The state-feedback controller gain K is chosen such that all the closed-loop poles are located at $-\omega_c$; $k_p = \omega_c^2$, $k_d = 2\omega_c$ [9, 15]. Substituting (11) into (10), the observer-based ADRC control law is given as:

$$u = \frac{(-z_3 + u_0)}{b} \quad (13)$$

which reduces the DC motor to a nominal double integral plant:

$$\ddot{y} = u_0 \quad (14)$$

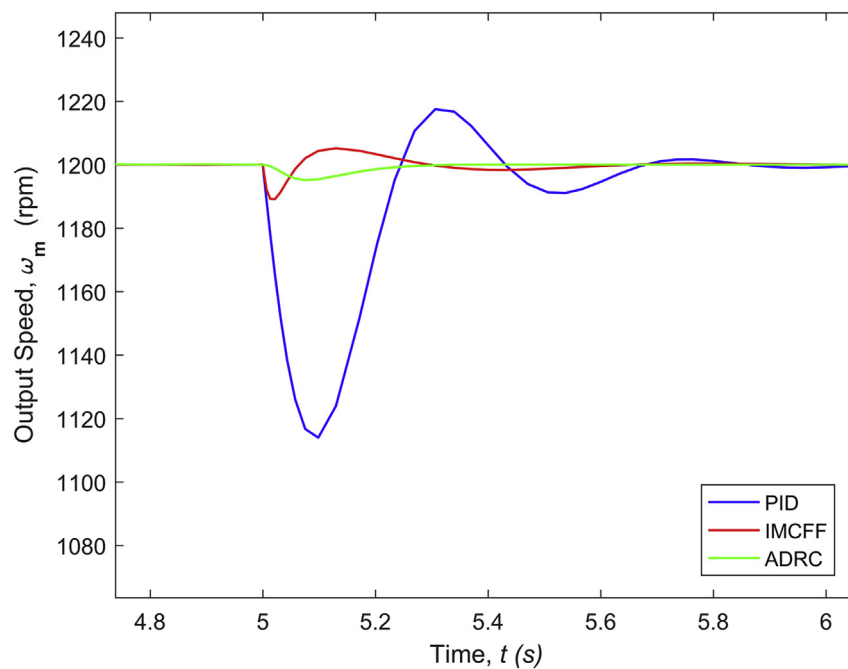


Figure 4. Magnified View of the Regulatory Control Performance of the ADRC controller.

Table 1. Coefficients of the observer-based controller.

Coefficients of $G_c(s)$	Formulae
N_0	$\omega_c^2 \omega_0^3$
N_1	$3\omega_c^2 \omega_0^2 + 2\omega_c \omega_0^3$
N_2	$3\omega_c^2 \omega_0 + 6\omega_c \omega_0^2 + \omega_0^3$
D_0	$\omega_c^2 + 3\omega_0^2 + 6\omega_c \omega_0$
D_1	$2\omega_c + 3\omega_0$
D_2	1

Thus, there are only two parameters to tune for the ADRC controller: L , the observer gain for the ESO, and the controller gain K . The primary tuning parameters for this controller are ω_c the controller bandwidth and ω_o the observer bandwidth. Higher values of ω_c and ω_o will result in improved disturbance-rejection performance and a more robust tracking performance [20]. Hence, with this Linear Extended State Observer (LESO), robust tracking performance and disturbance rejection are achieved. The block diagram of the observer-based ADRC control is given in Figure 1.

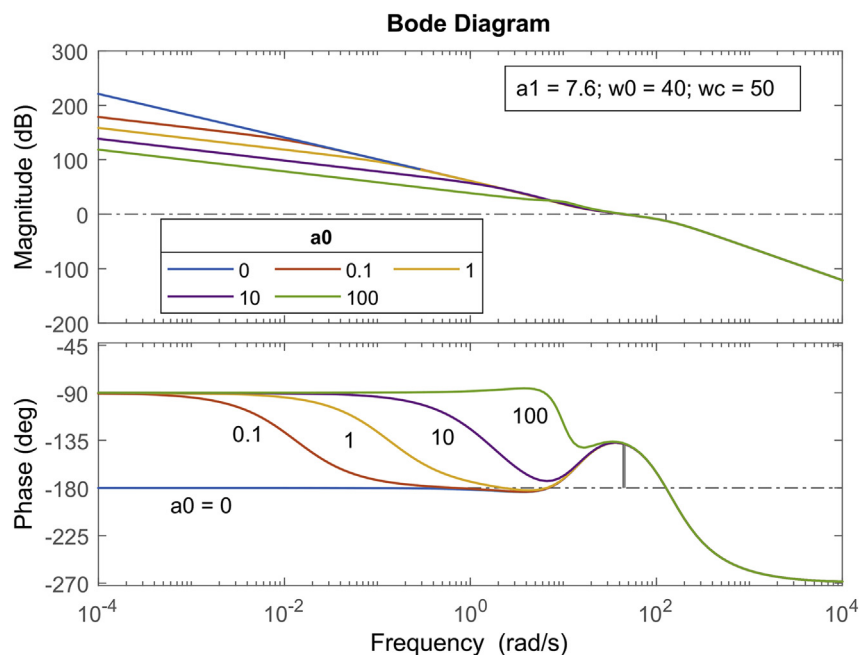


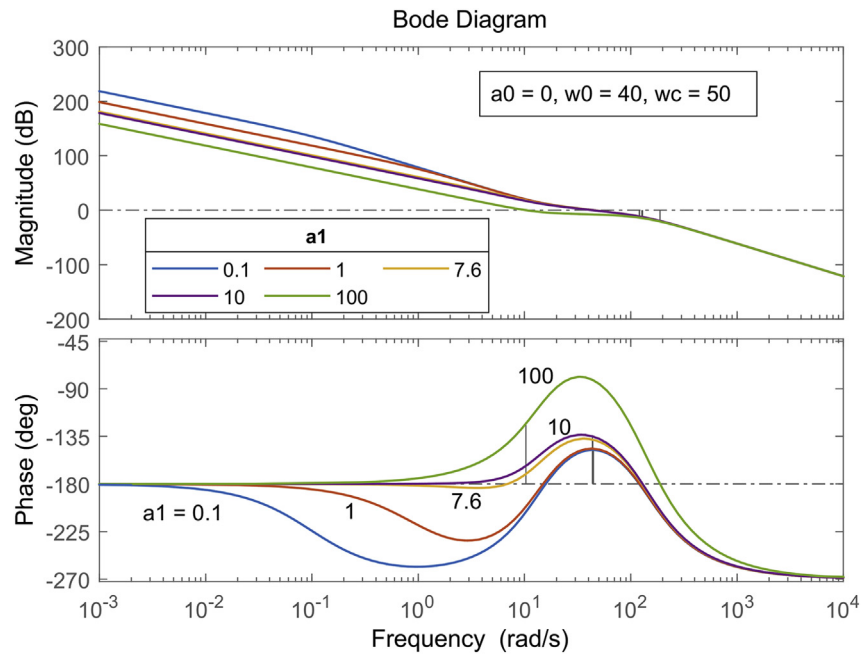
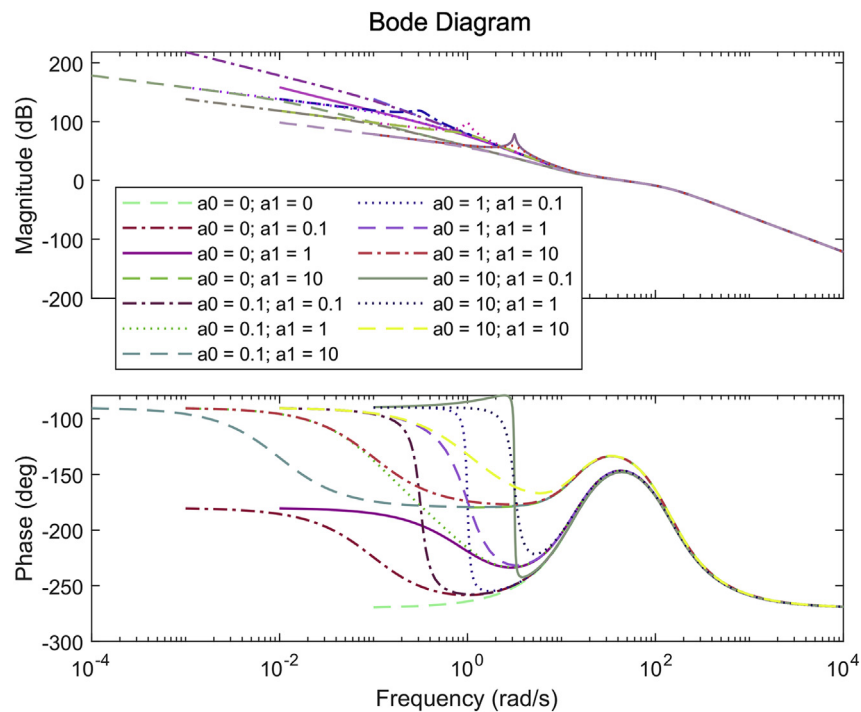
Figure 5. Loop gain Bode plots for different values of a_0 .

Table 2. Gain and Phase Margins at different values of a_0

a_0	Gain Margin (dB)	Phase Margin (deg)
0	12.3488	41.8914
0.1	12.3487	41.8915
1	12.3483	41.8919
10	12.3439	41.8951
100	12.3003	41.8764

Table 3. Gain and Phase Margins at different values of a_1

a_1	Gain Margin (dB)	Phase Margin (deg)
0.1	11.4573	32.2026
1	11.5672	33.3607
7.6	12.3488	41.8914
10	12.6236	45.0200
100	20.8231	57.0532

**Figure 6.** Loop gain Bode plots for different values of a_1 .**Figure 7.** Loop gain Bode plots for different values of a_0 and a_1 .

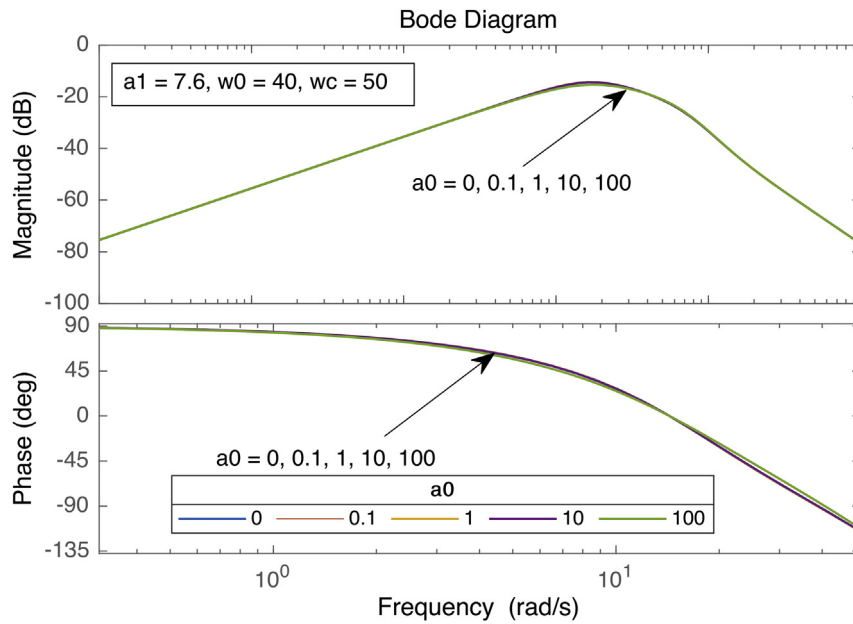


Figure 8. Bode plots of $G_{dy}(s)$ for $a_1 = 7.6$ and $a_0 = [0, 0.1, 1, 10$ and $100]$.

3. Simulation

Considering the motion control example, given by a second-order separately-excited DC motor

$$\ddot{y} = -7.6\dot{y} - 97.39y + 142.94u - T_d \quad (15)$$

Where $b = 142.94$, T_d is the load torque which constitutes a disturbance to the DC motor, and a LESO ADRC designed with $\omega_o = 40 \text{ rad/sec}$, $\omega_c = 50 \text{ rad/sec}$. The reference speed ω_{ref} levels for the motor is chosen to be the same as in [9]. As stated earlier, the servo performance of the ADRC controller is compared with that of the Feedback-Feedforward compensator in [1] and conventional PID control. The servo-control performance of the ADRC controller is given in Figure 2. From the resultant response, it can be seen that the observer-based controller

produces a robust reference tracking performance compared to the other control schemes. Its response is also non-oscillatory and damped.

A similar disturbance of 40 Nm as in [9], representing the load torque T_d is applied to the DC motor after 5 seconds, at constant reference speed. The regulatory control performance of the ADRC controller is shown in Figure 3, with the magnified version in Figure 4.

From the derived response, it is seen that the observer-based control scheme is more robust and possesses better disturbance rejection properties compared to the other control schemes.

3.1. Frequency response analysis

The DC motor being considered is equivalent to a linear time-invariant second-order plant given by:

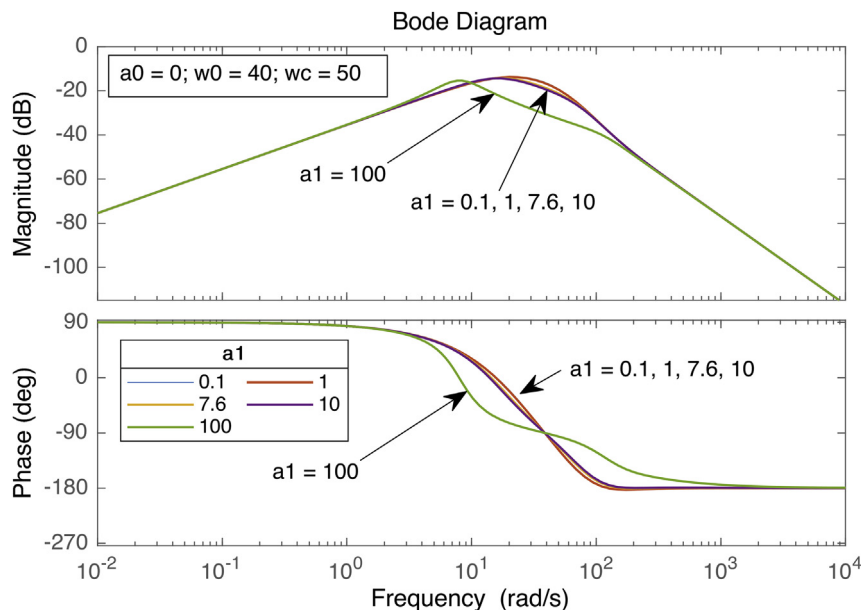


Figure 9. Bode plots of $G_{dy}(s)$ for $a_0 = 0$ and $a_1 = [0.1, 1, 7.6, 10$ and $100]$.

$$\ddot{y} = -a_1\dot{y} - a_0y + bu \quad (16)$$

Since both the DC motor and the LESO-ADRC controller are linearized, the robustness of the control system is evaluated using frequency analysis [21]. From the regulatory control performance, if the ADRC indeed estimates $\hat{h}$ and cancels it out, there should be a very minute change in bandwidth and stability margins when a_0 and a_1 vary.

The frequency analysis is carried out using the Bode plots of the loop gain, $G_l(s)$ and disturbance transfer functions, $G_{dy}(s)$ given in (17) and (18), respectively.

$$G_l(s) = G(s)G_c(s) \quad (17)$$

$$G_{dy}(s) = \frac{G(s)}{1 + G(s)G_c(s)} \quad (18)$$

Where $G(s)$ is the transfer function of the LTI plant in (16) given by:

$$G(s) = \frac{142.94}{s^2 + 7.6s + 97.39} \quad (19)$$

And $G_c(s)$ is the transfer function of the observer-based controller given by:

$$G_c(s) = \frac{1}{bs} \left(\frac{N_2s^2 + N_1s + N_0}{D_2s^2 + D_1s + D_0} \right) = \frac{844000s^2 + 1.84 \times 10^7s + 1.6 \times 10^8}{140s^3 + 30800s^2 + 2.702 \times 10^6s} \quad (20)$$

The values of N_2 , N_1 , N_0 , D_2 , D_1 , and D_0 in Eq. (20) are computed using the formulae shown in Table 1 [22].

The Bode plot for the loop gain transfer function is shown under different conditions in Figure 5, with $\omega_c = 50 \text{ rad/sec}$, $\omega_0 = 40 \text{ rad/sec}$, $b = 140$, $a_1 = 7.6$ and while $a_0 = [0, 0.1, 1, 10, 100]$. Table 2 summarizes the corresponding gain and phase margin values for this case. As can be seen from Table 2, there is virtually no change in the gain and phase margins with parameter variations.

The Bode diagrams of the loop transfer function for $a_0 = 0$, and $a_1 = 0.1, 1, 7.6, 10, 100$ is shown in Figure 6. Similarly, Table 3 shows the stability margins for each curve in Figure 6. As expected, the gain and phase margins are just as insensitive to changes in a_1 as to those in a_0 ; they are fairly constant, even with model parameter variations.

In Figure 7, the Bode diagrams for the case where both a_0 and a_1 are varied simultaneously is illustrated. A satisfactory insensitivity to changes in the parameters is observed from the plots.

The Bode diagrams for the transfer function between the input disturbance and output, $G_{dy}(s)$ are shown in Figures 8 and 9. In Figure 8, a_1 is held constant at 7.6, while a_0 is varied. As can be seen, the plots are overlapping indicating a constant gain margin and phase margin, even with changes in the plant parameters.

In Figure 9, a_0 is held constant at a value of 0, while a_1 is varied between the values of 0.1, 1, 7.6, 10, and 100. Likewise, the gain and phase margins of the Bode plots are reasonably constant even with the inconsistencies in the model. This outcome implies that consistent input disturbance rejection is realized even with the incidence of significant uncertainties of a_0 and a_1 .

4. Conclusion

The observer-based ADRC technique provides an effective control structure for the sampled DC motor where robust tracking performance, active disturbance rejection, and minimal modeling uncertainty are achieved. The linear extended state observer (LESO) in real-time estimates the general disturbance – consisting of the external interference and internal uncertainties – and feeds it back in the control loop thus achieving a robust and stable performance.

There are only two key parameters to tune for the ADRC controller: the observer bandwidth and the controller bandwidth, where higher values of both will result in improved disturbance-rejection performance and more rigorous control action. The frequency response analysis shows that this controller significantly enhances the stability margin and robustness of the machine in the incidence of parametric uncertainties. Remarkably, there is a high level of consistency in the bandwidth and stability margins of the DC motor in the face of substantial parametric changes. This phenomenon is likewise observed with its disturbance rejection performance. Hence, it can be held that even if the DC motor is unstable, stability can be guaranteed for the closed-loop system, with an increase in the observer and controller bandwidths. The implication is that the higher the bandwidth, the more parametric uncertainties the DC motor can tolerate.

Finally, the observer-based controller idealizes the complex DC motor plant by getting rid of all its imperfections. Future research on this work will focus on the implementation of the ADRC approach on the DC motor using a 3D representation, applying a nonlinear observer-based ADRC controller on the nonlinear DC machine, and implementing the control approach on a Permanent Magnet Synchronous Motor (PMSM).

Declarations

Author contribution statement

Ihechiluru Samuel Okoro: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Clinton O. Enwerem: Performed the experiments; Analyzed and interpreted the data.

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Data availability statement

No data was used for the research described in the article.

Declaration of interests statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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